

Closing Thurs: HW 9.6, 9.7(1)

Read HW hints in newsletter!!!!

Closing Tues: HW 9.7(2), 9.8, 9.9

Exam 1 is Thurs, Jan. 25th covers 9.4 - 9.9.

Derivative Rules: Sum, Coef. Rules and

$$\frac{d}{dx}(x^n) = n x^{n-1}.$$

$$\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Entry Task: Consider the function

$$f(x) = (x^3 - 2x + 1)(3 + 5x - 6\sqrt{x})$$

- Find $f'(x)$.
- Find the *height* at $x = 1$.
- Find the *slope* at $x = 1$.
- Give the *equation of the tangent* line at $x = 1$.

$$f(x) = (x^3 - 2x + 1)(3 + 5x - 6x^{1/2})$$

$$\begin{aligned} (a) f'(x) &= (x^3 - 2x + 1)(5 - 3x^{-1/2}) + (3x^2 - 2)(3 + 5x - 6x^{1/2}) \\ &= (x^3 - 2x + 1)\left(5 - \frac{3}{\sqrt{x}}\right) + (3x^2 - 2)(3 + 5x - 6\sqrt{x}) \end{aligned}$$

$$(b) \text{ "HEIGHT AT } x=1 \text{ " } = f(1)$$

$$\begin{aligned} &= ((1)^3 - 2(1) + 1)(3 + 5(1) - 6\sqrt{1}) \\ &= (1 - 2 + 1)(3 + 5 - 6) \\ &= 0 \cdot 2 = 0 \end{aligned}$$

$$(c) \text{ "SLOPE AT } x=1 \text{ " } = f'(1)$$

$$\begin{aligned} &= ((1)^3 - 2(1) + 1)\left(5 - \frac{3}{1}\right) + (3(1)^2 - 2)(3 + 5(1) - 6(1)) \\ &= (0)(2) + (1)(2) = 2 \end{aligned}$$

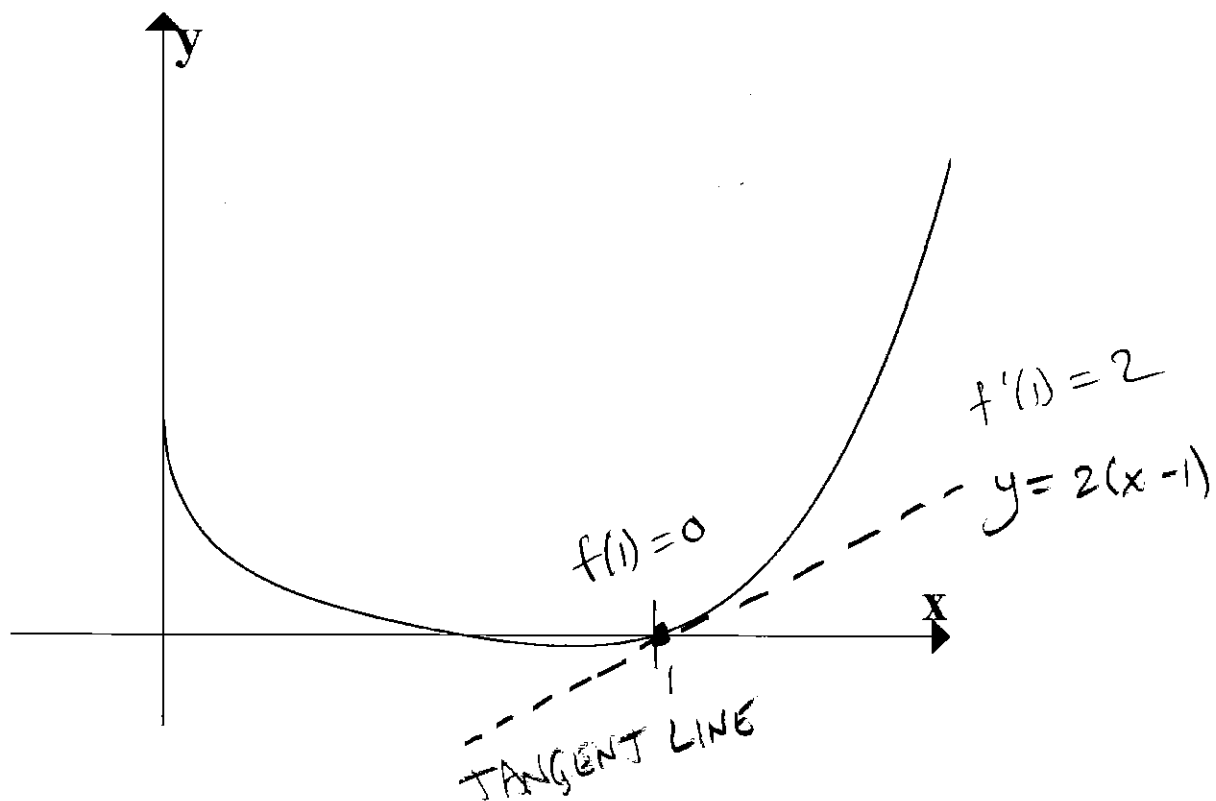
(d)

$$y = 2(x - 1) + 0$$

slope \swarrow x -coord \downarrow height \swarrow

$$\text{or } y = 2x - 2$$

$$f(x) = (x^3 - 2x + 1)(3 + 5x - 6\sqrt{x})$$



9.7 Chain Rule / Combining Rules

Generalized Power rule:

$$\frac{d}{dx} \left((g(x))^n \right) = n(g(x))^{n-1} g'(x)$$

CHAIN RULE:

$$\frac{d}{dx} (f(g(x))) = f'(g(x))g'(x)$$

Ex $y = (3x^2 - 4x)^{10}$ $u = g(x) = 3x^2 - 4x = \text{INSIDE} \rightarrow g'(x) = 6x - 4$
 $f(u) = u^{10} = \text{OUTSIDE} \rightarrow f'(u) = 10u^9$

$$y' = 10u^9 \cdot (6x - 4) = 10(3x^2 - 4x)^9 \cdot (6x - 4)$$

Ex $y = \sqrt{5x^2 + x^3} = (5x^2 + x^3)^{1/2}$

$$y' = \frac{1}{2} (5x^2 + x^3)^{-1/2} \cdot (10x + 3x^2) = \frac{10x + 3x^2}{2\sqrt{5x^2 + x^3}}$$

Ex $y = \frac{10}{(x^5 + 2x^4)^6} = 10(x^5 + 2x^4)^{-6}$

$$y' = -60(x^5 + 2x^4)^{-7} (5x^4 + 8x^3) = \frac{-60(5x^4 + 8x^3)}{(x^5 + 2x^4)^7}$$

Combining Rules

Step 0: Simplify and rewrite powers.

$$\frac{1}{x^r} = x^{-r}, \quad \sqrt[n]{x} = x^{1/n}$$

Step 1: Identify overall form

Sum: $A + B$

Product: $F \cdot S$

Quotient: $\frac{N}{D}$

Chain: $(\text{inside})^n$

$$\overbrace{(x^2+1)^3}^A + \overbrace{5x}^B$$

$$\frac{x^5}{F} \cdot \overbrace{(x^3+1)^{10}}^S$$

$$\frac{\sqrt{x}}{x^2+3} \leftarrow \begin{matrix} N \\ D \end{matrix}$$

$$(x^7 + x^5)^{11} \leftarrow (\text{inside})^{11}$$

Step 2: Apply rule.

As part of that rule, you likely will have to do more derivative. For those derivatives go to step 1.

Practice: Find the derivatives

Several of these are directly from HW!

$$1. y = \frac{5\sqrt{1-x^3}}{9}$$

$$y = \frac{5}{9} (1-x^3)^{1/2} \quad \leftarrow \text{(inside)}^{1/2}$$

$$y' = \frac{5}{9} \cdot \frac{1}{2} (1-x^3)^{-1/2} \cdot (-3x^2)$$

$$= -\frac{5}{9} \cdot \frac{3}{2} \frac{x^2}{\sqrt{1-x^3}} = -\frac{5}{6} \frac{x^2}{\sqrt{1-x^3}}$$

$$2. y = \frac{(x^2+2)^3}{x^4+5x} \leftarrow N \leftarrow D$$

$$N = (x^2+2)^3 \rightarrow N' = 3(x^2+2)^2(2x)$$

$$D = x^4+5x \rightarrow D' = 4x^3+5$$

$$\frac{DN' - ND'}{D^2} = \frac{(x^4+5x) \cdot 3(x^2+2)^2(2x) - (x^2+2)^3(4x^3+5)}{(x^4+5x)^2}$$

$$= \frac{(x^2+2)^2 [6x(x^4+5x) - (x^2+2)(4x^3+5)]}{(x^4+5x)^2}$$

$$3. y = \left(\frac{2x - 4}{x^3 + 1} \right)^5$$

$$y' = 5 \left(\frac{2x-4}{x^3+1} \right)^4 \cdot \left(\frac{(x^3+1)(2) - (2x-4)(3x^2)}{(x^3+1)^2} \right)$$

$$4. y = \frac{5}{x^3} + 6x^2 \sqrt{x^5 + 1}$$

$$y = 5x^{-3} + \underbrace{6x^2}_F \underbrace{(x^5+1)^{1/2}}_S$$

$$y' = -15x^{-4} + 6x^2 \cdot \frac{1}{2} (x^5+1)^{-1/2} (5x^4) + 12x (x^5+1)^{1/2}$$

$$\uparrow$$

$$-\frac{15}{x^4}$$

$$\uparrow$$

$$\frac{1}{\sqrt{x^5+1}}$$

$$5. y = \frac{7}{2(x^4 + 8)^5} - 5x + 4$$

$$y = \frac{7}{2} (x^4 + 8)^{-5} - 5x + 4$$

$$y' = -\frac{35}{2} (x^4 + 8)^{-6} \cdot 4x^3 - 5$$

$$6. y = \underbrace{(t^2 + 4)^5}_F \underbrace{(t^3 - 2)^4}_S$$

$$y' = (t^2 + 4)^5 \cdot 4(t^3 - 2)^3 \cdot 3t^2 + 5(t^2 + 4)^4 (2t) (t^3 - 2)^4$$

9.8 Second Derivative

The *second derivative* is the derivative of the derivative. We denote it

$$f''(x) \quad \text{or} \quad \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2y}{dx^2}$$

Example:

$$y = x^3 + 2x$$

$$y' = 3x^2 + 2$$

$$y'' = 6x$$

The second derivative represents the rate at which the *rate* of the original quantity is changing.

(*rate of change of rate of change*)

We will interpret what this means more later, for now compute it.

Ex) $y = (x^3 + 2)^4$

$$y' = 4(x^3 + 2)^3 \cdot 3x^2 = \underbrace{12x^2}_F \underbrace{(x^3 + 2)^3}_S$$

$$y'' = 12x^2 \cdot 3(x^3 + 2)^2 \cdot 3x^2 + 24x(x^3 + 2)^3$$

Example: Suppose

$$R(x) = 70x + 0.4x^3$$

is the revenue (in dollars) if you sell x items.

1. What is the marginal revenue (denoted MR or \overline{MR}) when you sell 10 items?

change to 10 in lecture

$$MR(x) = R'(x) = 70 + 1.2x^2$$

$$R'(10) = 70 + 1.2(10)^2$$

$$= 70 + 1.2(100)$$

$$= 70 + 120$$

$$= 190 \text{ \$/item}$$

(Next item will bring in about \$190 in revenue)

2. What is the rate of change of marginal revenue when you sell 10 items?

$$MR'(x) = R''(x)$$

$$= 2.4x$$

$$MR'(10) = 2.4(10) = 24 \frac{\text{\$/item}}{\text{item}}$$

MR will go up about 24 $\text{\$/item}$ over the next item.

MR is increasing
 R is "speeding up"
(accelerating)